Deep Combination of CDCL(T) and Local Search for Satisfiability Modulo Non-Linear Integer Arithmetic Theory

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Outlin

- **Backgrounds**
- Two-Layer Hybrid Framework
- Experiments
- Conclusions

SMT(NIA)

The Satisfiability Problem (SAT)

Propositional Logic

SAT

generalize

FOL Background Theories

- Bit-vectors
- Linear Real Arithmetic
- Non-Linear Integer Arithmetic (NIA)
- … …

Satisfiability Modulo Theory (SMT)

First-Order Logic (FOL)

SMT

$$S_F = (p_1 \vee \neg p_2) \wedge (\neg p_{\sigma_1} \vee p_{\sigma_2}) \wedge (p_2 \vee \underline{p_{\sigma_3}})$$

Boolean encoder

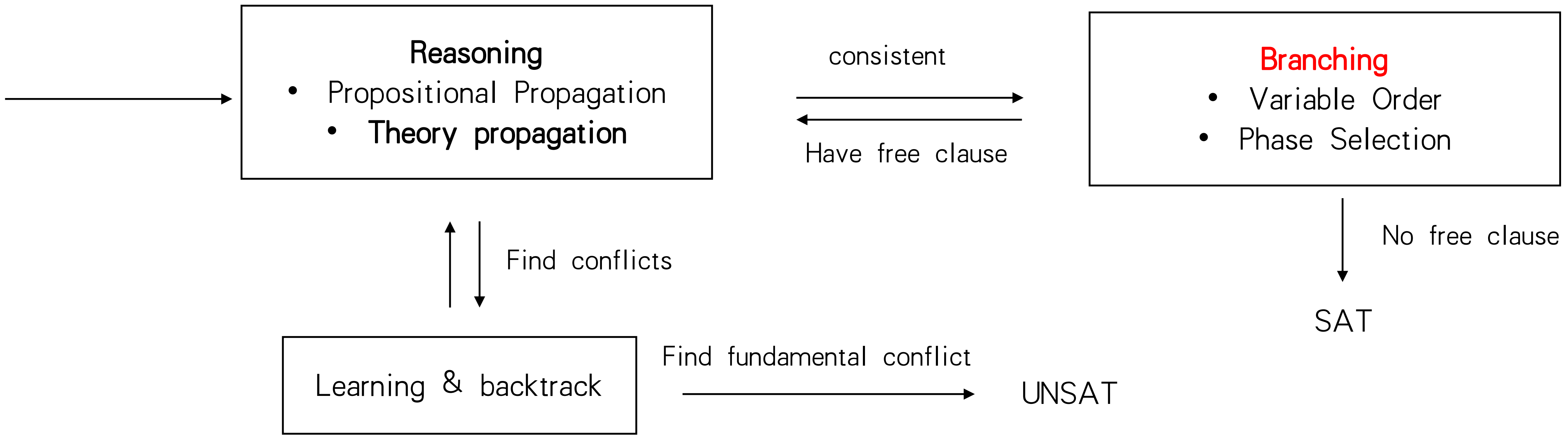
skeleton

$$\begin{aligned} F_{SMT(NIA)} &= (p_1 \vee \neg p_2) \\ &\wedge (\neg(3x_1x_2 \leq 2) \vee (-x_2 - 3x_4 \leq 0)) \\ &\wedge (p_2 \vee \underline{(2x_3^3x_4 + 4x_2 + x_1 \leq 8)}) \end{aligned}$$

Arithmetic literal

Algorithms for SMT

- Eager Method: Bit–blasting
- Lazy Method: CDCL(T)
- Model–Construction Satisfiability (MCSat) calculus
- Stochastic Local Search (SLS)
- … …



CDCL(T) flow

Success for Local Search

Portfolios with SLS won recent arithmetic tracks

Finding a solution

Cai S., Li B., Zhang X. Local Search for SMT on Linear Integer Arithmetic (CAV 22)
Cai S., Li B., Zhang X. Local Search For Satisfiability Modulo Integer Arithmetic Theories (TOCL 23)
Li B., Cai S. Local Search for SMT on Linear and Multi-Linear Real Arithmetic (FMCAD 23)

Previous work for SAT

Our hybrid methods on SAT significantly improves CDCL with the help of SLS.

Ten Challenges in Propositional Reasoning and Search

7th: *Demonstrate the successful combination of stochastic search and systematic search techniques, by the creation of a new algorithm that outperforms the best previous examples of both approaches.*

--- Bart Selman, et al., AAAI, 1997

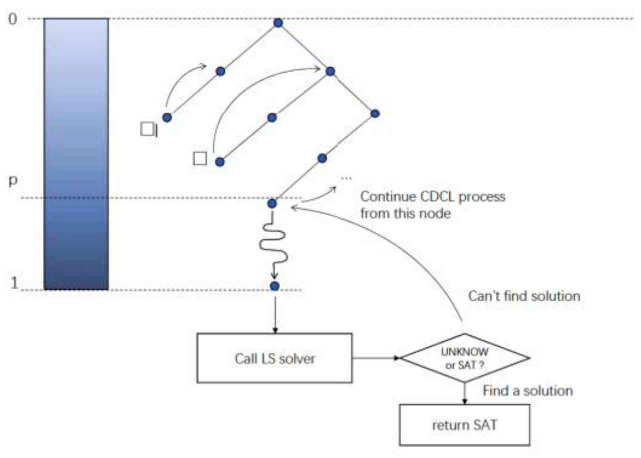


Fig. 1. Overall Procedure of Relaxed CDCL

SLS sampling ←→ CDCL solving
Boosting CDCL with SLS information

Plug SLS into a CDCL solver

- Calling SLS on promising branches
- Filter similar branches

S. Cai, X. Zhang: Deep Cooperation of CDCL and Local Search for SAT, *SAT* 2021 (best paper).

Motivation

SAT hybrid Methods --> SMT hybrid Methods ?

Outlin

- Backgrounds
- **Two-Layer Hybrid Framework**
  - Q1： *How to **schedule** SLS and CDCL(T) reasonably?*
  - Q2： *How to use CDCL(T) to **guide** SLS?*
  - Q3： *How to **exploit** information from SLS to enhance CDCL(T)?*
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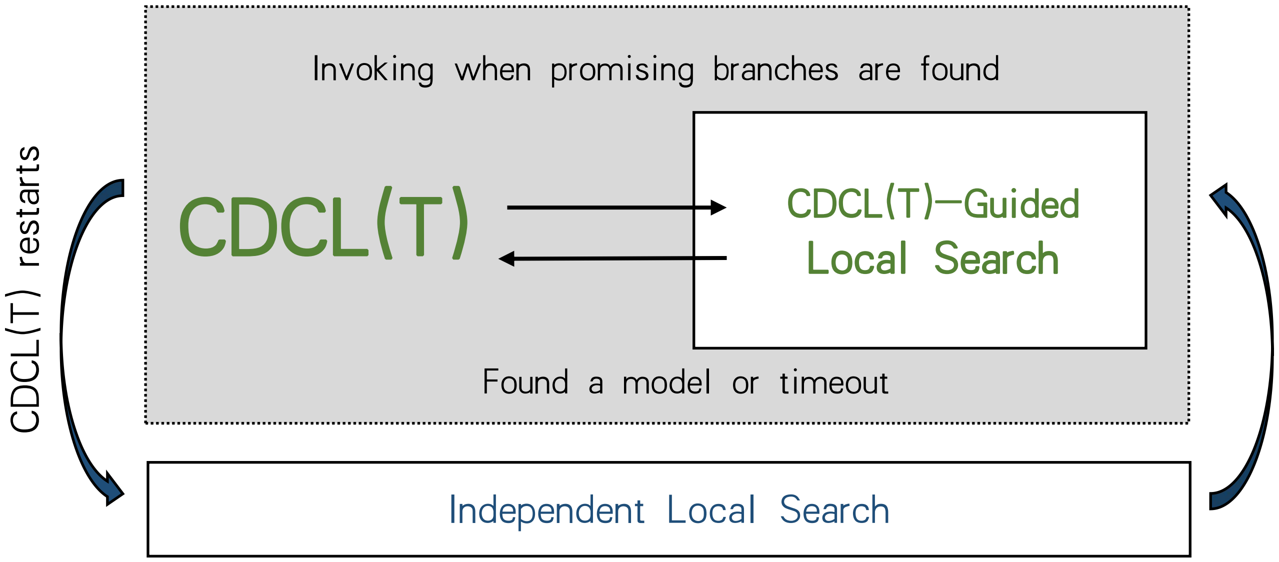
Two-layer Hybrid Framework

Q1： *How to **schedule** SLS and CDCL(T) reasonably?*



Inner：

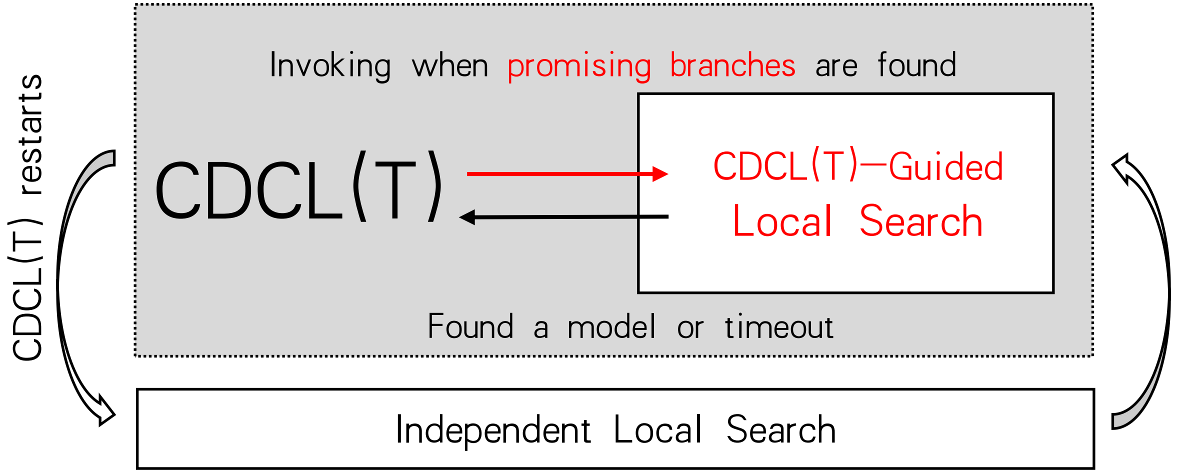
- CDCL(T) as the **main-body**
- CDCL(T)-guided SLS **sampling**

Outer：

- Portfolio CDCL(T) and Independent-SLS
- Increasing intervals

Two-Level Hybrid Framework

- Q2： *How to use CDCL(T) to **guide** SLS?*

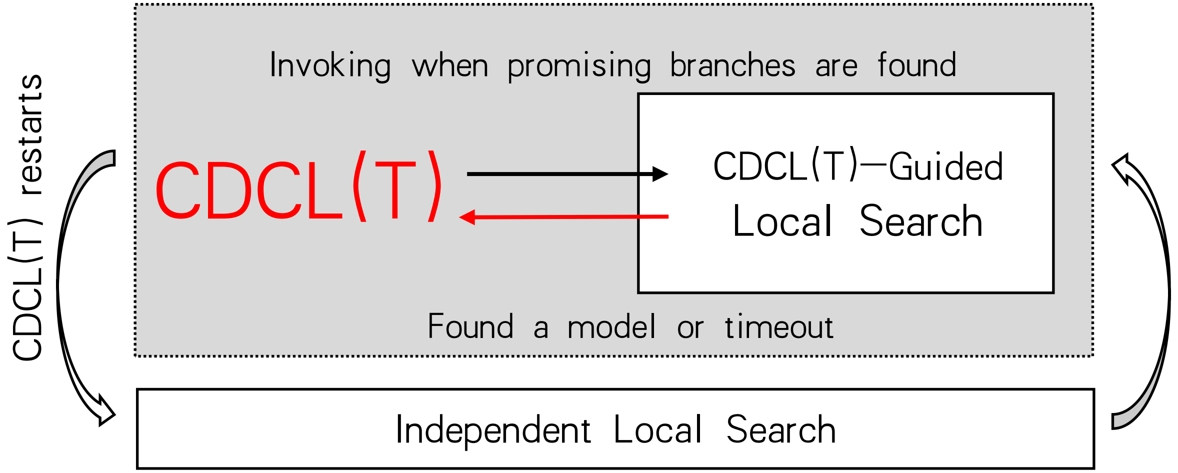


Promising Branch

- High consistency
  - Core—clauses are satisfied
  - Extracting a sub—formula
- Pruning similar branches
  - count the number of special backtracks

Two-Level Hybrid Framework

- Q3： *How to **exploit** information from SLS to enhance CDCL(T)?*



Phase resetting with SLS

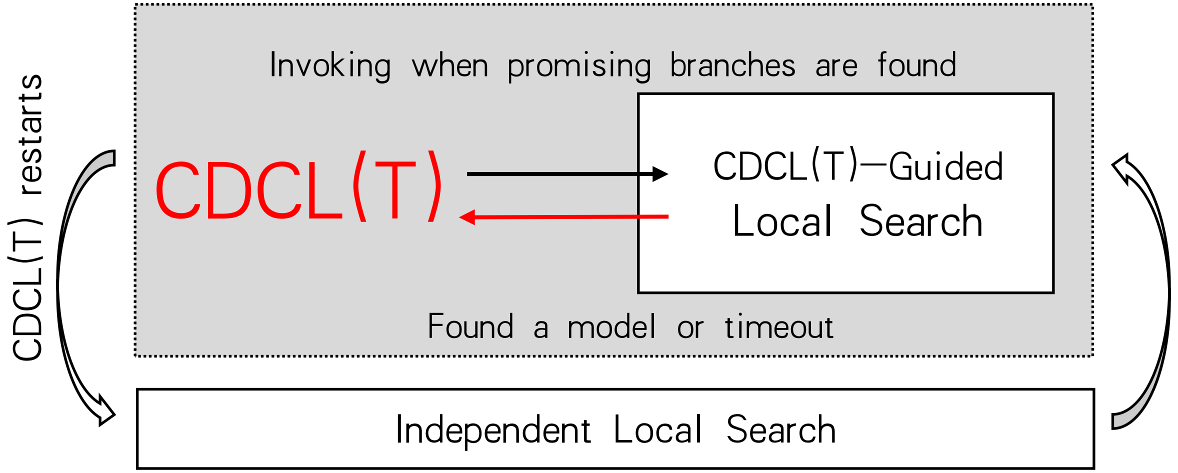
- Record the best assignment from the latest CDCL(T)—Guided SLS α_{sls}.
- Choose a phase according to probability distribution when restarts.

Phase Type	α_{sls}	α_T	α_F	α_{rnd}	α_{rev}	No Change
Probability	50%	10%	10%	10%	10%	10%

Intensification(50%) *Diversification(50%)*

Two-Level Hybrid Framework

- Q3： *How to **exploit** information from SLS to enhance CDCL(T)?*



Variable ordering with SLS

- **Conflict frequency** of a skeleton variable

$$S_F = (p_1 \vee \neg p_2) \wedge (\neg p_{\sigma_1} \vee p_{\sigma_2}) \wedge (p_2 \vee \underline{p_{\sigma_3}})$$

$$\begin{aligned} F_{SMT(NIA)} \quad = \quad & (p_1 \vee \neg p_2) \\ \wedge \quad & (\neg(3x_1x_2 \leq 2) \vee (-x_2 - 3x_4 \leq 0)) \\ \wedge \quad & \underline{(p_2 \vee (2x_3^3x_4 + 4x_2 + x_1 \leq 8))} \end{aligned}$$ Clause c

- falsified in 3 steps
- 10 steps in total
- $fc(p_{\sigma_3}) = \frac{3}{10}$
- branching score of p_{σ_3} add $a \cdot fc(p_{\sigma_3})$

Outlin

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- **Experiments**
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Experimen Compare with SOTA

↑1458(7.43%) ↑3207(17.95%) ↑4629(28.15%) ↑6811(47.76%) ↑3332(18.78%)

Benchmark	#Ins	VBS$_1$			HYBRIDSMT			Z3			MATHSAT5			YICES2			CVC5			NINC-CORES		
		#S	#U	#A	#S	#U	#A	#S	#U	#A	#S	#U	#A	#S	#U	#A	#S	#U	#A	#S	#U	#A
AProVE	2409	1663	733	2396	1652	696	2348	1658	697	**2355**	1647	565	2212	1593	**711**	2304	1373	622	1995	**1663**	245	1908
Calypto	177	80	97	177	78	**97**	175	**80**	95	175	79	90	169	79	96	175	79	94	173	**80**	**97**	**177**
CInteger	1818	863	707	1570	**857**	**686**	**1543**	771	510	1281	717	458	1175	520	463	983	323	378	701	849	140	989
Dartagnan	374	13	341	354	**13**	**341**	**354**	**13**	**341**	**354**	**13**	326	339	9	319	328	**13**	324	337	0	148	148
ezsmt	8	8	0	8	**8**	0	**8**	**8**	0	**8**	**8**	0	**8**	**8**	0	**8**	**8**	0	**8**	0	0	0
ITS	17046	9686	4712	14398	**9594**	**4613**	**14207**	8540	3977	12517	7784	3794	11578	6816	3557	10373	5502	3172	8674	9428	2314	11742
LassoRanker	106	4	102	106	4	**102**	**106**	4	**102**	**106**	4	101	105	4	85	89	4	93	97	4	89	93
LCTES	2	0	2	2	0	**2**	**2**	0	**2**	**2**	0	1	1	0	0	0	0	1	1	0	0	0
SAT14	1926	1853	73	1926	**1853**	**73**	**1926**	1852	68	1920	1801	67	1868	1840	66	1906	1802	72	1874	**1853**	63	1916
sqrtmodinv	27	0	11	11	0	**10**	**10**	0	**10**	**10**	0	0	0	0	0	0	0	1	1	0	0	0
ULasso	32	6	26	32	6	25	31	6	**26**	**32**	6	**26**	**32**	6	**26**	**32**	6	**26**	**32**	6	**26**	**32**
UAuto	7	0	7	7	0	**7**	**7**	0	**7**	**7**	0	**7**	**7**	0	**7**	**7**	0	**7**	**7**	0	1	1
UAuto23	58	8	13	21	7	3	10	2	2	4	7	**10**	**17**	5	0	5	**8**	6	14	5	0	5
Leipzig	167	160	4	164	157	2	159	**159**	1	**160**	128	2	130	101	1	102	90	2	92	155	**4**	159
Math	1100	687	7	694	100	**7**	107	**659**	**7**	**666**	203	**7**	210	112	**7**	119	227	**7**	234	550	0	550
MCM	186	84	5	89	**78**	0	**78**	15	1	16	13	0	13	11	0	11	16	**4**	20	19	0	19
ALL	25443	15115	6840	21955	14407	**6664**	**21071**	13767	5846	19613	12410	5454	17864	11104	5338	16442	9451	4809	14260	**14612**	3127	17739

Table 2: Compared to four state-of-the-art solvers shown in recent SMT-COMP and the best solver in [13]. VBS$_1$ is the virtual best solver of all the solvers presented in this paper.

HybridSMT significantly outperforms state-of-the-art SMT(NIA) solvers

Experimen Analysis for Base Solvers

Benchmark	HybridSMT			CDCL(T)			LocalSMT
	#S	#U	**#A**	#S	#U	**#A**	#S/ #A
AProVE	**1652**	**696**	**2348**	1643	695	2338	1620
CInteger	**857**	**686**	**1543**	717	681	1398	790
Dartagnan	**13**	**341**	**354**	**13**	**341**	**354**	0
ITS	9594	4613	**14207**	8273	**4638**	12911	**9627**
SAT14	**1853**	**73**	**1926**	**1853**	**73**	**1926**	1478
Math	**100**	**7**	**107**	**100**	**7**	**107**	0
MCM	78	0	78	15	**1**	16	**86**
Other	**260**	248	**508**	228	**249**	477	245
ALL	**14407**	6664	**21071**	12842	**6685**	19527	13846

Table 4: Effectiveness of the hybrid methods

Benchmark	For SAT instances									For UNSAT instances						
	#S	VBS_2(S)	#B(S)	#ByI	#ByG	#Ct(I)	#Ct(G)	T(I)%	T(G)%	#U	VBS_2(U)	#B(U)	#Ct(I)	#Ct(G)	T(I)%	T(G)%
AproVE	1654	1662	1	893	34	0.59	0.51	5.97	1.72	696	695	4	1.3	21.16	13.5	19.02
CInteger	857	835	25	618	203	1.79	33.74	49.89	9.0	688	681	10	4.02	293.0	43.88	32.36
Dartagnan	13	13	0	0	0	0.0	0.0	0.0	0.0	341	341	0	0.06	0.03	3.75	0.28
ITS	9590	10216	67	5726	1247	1.09	13.25	46.05	5.16	4612	4638	41	3.96	318.47	38.42	33.41
SAT14	1853	1853	0	411	144	1.11	1.98	42.15	37.9	73	73	0	1.62	46.66	29.62	53.27
Math	100	100	0	1	0	1.02	1.01	22.88	22.37	7	7	0	1.29	2.0	12.65	13.98
MCM	81	86	9	81	0	2.19	42.16	68.54	10.36	0	1	-	-	-	-	-
Other	260	257	5	122	50	1.18	13.6	24.18	9.34	247	249	2	0.77	1.48	36.36	6.96
ALL	14408	15022	107	7852	1678	1.09	11.63	40.71	9.42	6664	6685	57	3.34	253.44	34.4	29.32

Table 6: Analysis for the role of internal local search solver in HybridSMT. VBS_2 denotes the virtual best solver of Z3 and LocalSMT.

Further Improvements with portfolios

Z3 = CDCL (T) + Eager Method + NLSat + ⋯

Hybrid+Z3 = hybridSMT + Eager Method + NLSat + ⋯

Benchmark	Hybrid+Z3			HybridSMT			Z3		
	#S	#U	**#A**	#S	#U	**#A**	#S	#U	**#A**
AProVE	1655	**697**	2352	1652	696	2348	**1658**	**697**	**2355**
CInteger	854	**691**	**1545**	**857**	686	1543	771	510	1281
Dartagnan	13	341	354	13	341	354	13	341	354
ITS	9589	4606	14195	**9594**	**4613**	**14207**	8540	3977	12517
SAT14	**1853**	**73**	**1926**	**1853**	**73**	**1926**	1852	68	1920
Math	**677**	7	**684**	100	7	107	659	7	666
MCM	**82**	0	**82**	78	0	78	15	**1**	16
Other	259	247	506	**260**	**248**	**508**	259	245	504
ALL	**14982**	6662	**21644**	14407	**6664**	21071	13767	5846	19613

Table 3: Hybrid+Z3: Improve Z3 by HybridSMT.

↑ *573*

Future Work

- Hybrid methods for other SMT background theories
- Hybrid methods of SLS with MCSat … …

Thank You!

Q&A